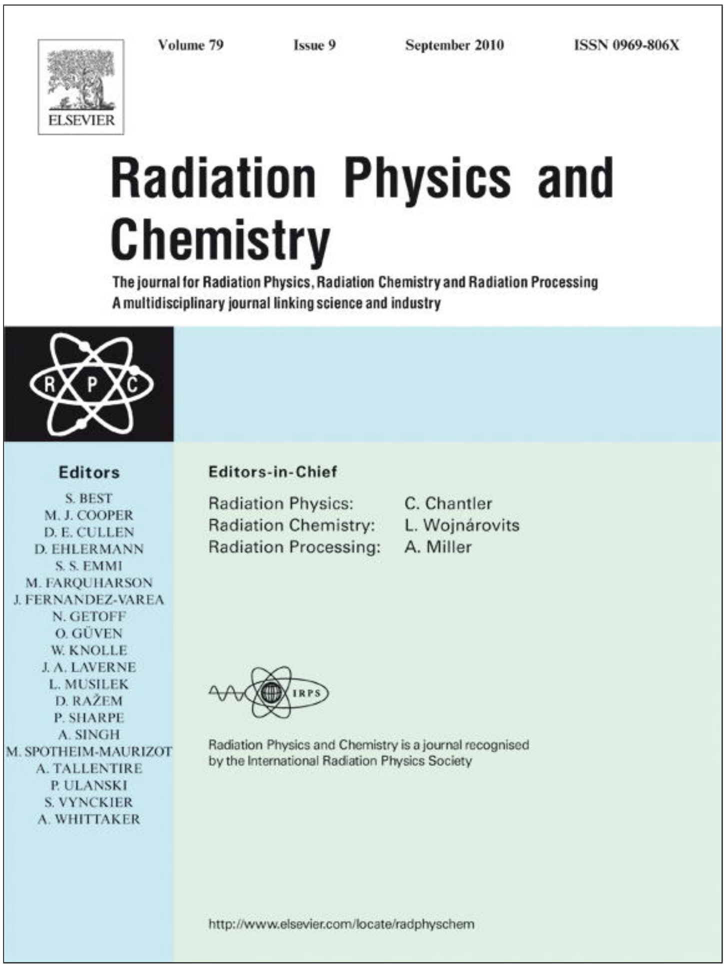
Volume 79
Issue 9
September 2010
ISSN 0969-806X
ELSEVIER
Radiation Physics and Chemistry
The journal for Radiation Physics, Radiation Chemistry and Radiation Processing
A multidisciplinary journal linking science and industry
RPC
Editors
S. BEST
M. J. COOPER
D. E. CULLEN
D. EHLERMANN
S. S. EMMI
M. FARQUHARSON
J. FERNANDEZ-VAREA
N. GETOFF
O. GÜVEN
W. KNOLLE
J. A. LAVERNE
L. MUSILEK
D. RAŽEM
P. SHARPE
A. SINGH
M. SPOTHEIM-MAURIZOT
A. TALLENTIRE
P. ULANSKI
S. VYNCKIER
A. WHITTAKER
Editors-in-Chief
Radiation Physics: C. Chantler
Radiation Chemistry: L. Wojnárovits
Radiation Processing: A. Miller
IRPS
Radiation Physics and Chemistry is a journal recognised by the International Radiation Physics Society
http://www.elsevier.com/locate/radphyschem

Radiation Physics and Chemistry 79 (2010) 929–932

Contents lists available at ScienceDirect

Radiation Physics and Chemistry

journal homepage: www.elsevier.com/locate/radphyschem

On the efficiency of azimuthal and rotational splitting for Monte Carlo simulation of clinical linear accelerators

L. Brualla *, W. Sauerwein

NCTeam, Strahlenklinik, Universitätsklinikum Essen, Hufelandstr. 55, D-45122 Essen, Germany

ARTICLE INFO

Article history:
Received 2 September 2009
Accepted 31 March 2010

Keywords:
Monte Carlo
Linac
Variance-reduction
Splitting
Radiotherapy

ABSTRACT

This article presents rotational splitting as a modification to the sampling process of the azimuthal angle used in the variance-reduction technique of azimuthal particle redistribution, with the goal to improve the efficiency of this variance-reduction technique for the Monte Carlo simulation of radiation transport in clinical linear accelerators. Using a constant azimuthal angle, instead of a random one, in the azimuthal particle redistribution technique, increases the efficiency of the simulation of a clinical linear accelerator by about 30% and reduces the latent variance of a $10\times10\,\text{cm}^2$ phase space by about 40%.

1. Introduction

With the advent of multiple core processors and the availability of faster computers, it is now feasible to perform Monte Carlo (MC) simulations of clinical linear accelerators (linacs) in a matter of hours (Reynaert et al., 2007; Chetty et al., 2007). By simulating a whole linac with a general-purpose MC radiation transport program, it is possible to avoid hybrid MC codes that employ analytical approximations and pre-calculated phase-space files (PSF). In order to increase the efficiency of MC simulations of linacs, Bush et al. (2007) devised a method based on the well-known variance-reduction technique of particle splitting. The application of their method, called azimuthal particle redistribution, allows a reduction of the latent variance of a phase space (Sempau et al., 2001) by more than a factor of 20, for a field size of $10\times10\,\text{cm}^2$, compared to the application of standard particle splitting. Azimuthal particle redistribution is particularly useful for the simulation of small radiation fields currently employed in radiation oncology with intensity-modulated radiotherapy, stereotactic radiotherapy or for specific treatments such as the irradiation of eye tumours (Brualla et al., 2009a).

A variation of the original azimuthal particle redistribution method is introduced in this article. The method presented, which will be referred to as rotational splitting, increases even further the efficiency of the azimuthal particle redistribution method.

2. The different particle splitting methods

Standard particle splitting is a variance-reduction technique in which a particle that has a significant probability of contributing to the final absorbed dose is replicated N times with each of its replicas having a statistical weight equal to $1/N$. Consequently, if the original particle is absorbed somewhere in the geometry of the linac before contributing to the final absorbed dose, its information is not completely lost because the replicas can still reach the final scoring zone.

Azimuthal particle splitting (Bush et al., 2007) is based on the same principle of standard splitting but can only be applied when the radiation beam and geometry are cylindrically symmetric, which is the case of the upper part of most linacs used in radiotherapy. In azimuthal particle splitting, each replica is rotated about the central beam axis. The same splitting number N is used for every split particle, independent of the distance of the original particle from the beam axis. The density of split particles varies inversely with the distance to the central axis and their statistical weight is kept constant. The azimuthal angle of each split particle is a random variable φ' $(0\le\varphi'<2\pi)$. Particle direction cosines, u and v, are transformed in order to conserve the direction of the original particle relative to the central beam axis, according to

$$\begin{pmatrix} u' \\ v' \end{pmatrix} = \begin{pmatrix} \cos(\phi'-\phi) & -\sin(\phi'-\phi) \\ \sin(\phi'-\phi) & \cos(\phi'-\phi) \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} \qquad (1)$$

where φ is the azimuthal angle of the original particle.

In the implementation of azimuthal splitting presented here, which will be referred to as rotational splitting, the azimuthal angle between two neighboring split particles is constant $(\Delta\phi = 2\pi/N)$, in contrast with the original azimuthal splitting

* Corresponding author. Tel.: +49 201 723 3488; fax: +49 201 723 5908.
E-mail addresses: lorenzo.brualla@uni-duisburg-essen.de, lbrualla@gmail.com (L. Brualla).

0969-806X/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.radphyschem.2010.03.020

technique of Bush et al. (2007), in which the azimuthal angle is a uniform random variable. Using constant azimuthal angles allows for a faster calculation of each shower and reduces statistical noise. The increase in efficiency, when using a constant azimuthal angle, is due to the smaller number of calculations required and the lower statistical noise introduced in the simulation, since less random numbers are required at every Monte Carlo step for estimating the same quantity. In other words, a quantity with zero variance, which is the constant azimuthal angle for the rotational splitting technique, is used instead of a quantity with finite variance, which is the random azimuthal angle used for the case of azimuthal splitting.

3. Efficiency study of azimuthal and rotational splitting

The efficiency of azimuthal and rotational splitting is studied by estimating with a MC algorithm the area enclosed by two parallel and symmetric chords of a circumference and the circumference itself, as shown in Fig. 1. The width of the area to be calculated is $2d$. This area can be calculated analytically with the formula

$$A = r^2[\pi-(\theta-\sin\theta)] \tag{2}$$

where r is the radius of the circumference and $\theta = 2\arccos(d/r)$. For simplicity, r has been set equal to 1.

For estimating the area A with a MC algorithm, random numbers are sampled inside a circumference using polar coordinates (R, φ) according to the distributions

$$\begin{aligned} R &= \sqrt{r^2\xi_1} \\ \phi &= 2\pi\xi_2 \end{aligned} \tag{3}$$

where ξ_1 and ξ_2 are two uniformly distributed random numbers and $r=1$ the radius of the circumference. Sampled points (R, φ) lie inside the area A if

$$|R\sin\phi| < d \tag{4}$$

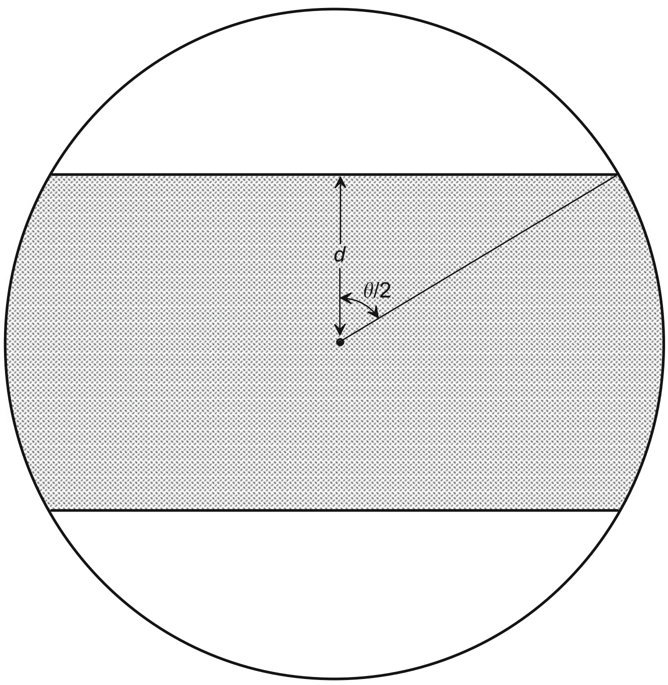

Fig. 1. The shaded region represents the area integrated with MC methods. The area varies with d.

It can be seen that this algorithm yields exactly $A=\pi$ for $d=r=1$ and $A=0$ for $d=0$.

Sampling of points using this algorithm resembles particles coming into a circular phase space of a linac where standard, azimuthal or rotational splitting can be applied. This analytical problem allows for the study of the intrinsic efficiency of the different splitting techniques since no computation time is employed in calculations other than the ones related to the splitting algorithms. This is not the case in the simulation of a linac in which most of the calculations are dedicated to the physical interactions and the transport algorithms through the geometry. A MC code that estimated the area shown in Fig. 1 using the algorithm described in Eqs. (3) and (4) was written. The code estimated the area using the following three methods: (i) no splitting, (ii) azimuthal splitting and (iii) rotational splitting. The efficiency of each evaluated method has been determined by

$$\varepsilon = \left(\frac{\bar{A}}{\sigma_A}\right)^2 \frac{1}{t} \tag{5}$$

where $\bar{A}$ is the MC estimated area, σ_A is the statistical uncertainty associated with the area and t is the CPU time employed in the calculation. All simulations presented here have been run for sufficiently long times in order to stabilize the efficiency.

The dependence of the statistical uncertainty of the area (σ_A), associated with each considered method, on the size of the area to be MC integrated has been studied by varying d from 0 to 1 in steps of 0.001 and using a constant splitting number of 15 in methods (ii) and (iii). Fig. 2 shows this dependence, where each value shown was obtained with 10^7 simulated histories. It can be observed that the three methods have the correct limit behaviour, that is $\lim_{d\to 0; d\to 1}\sigma_A = 0$. Use of either form of particle splitting produces a reduction of the statistical uncertainty, with rotational splitting producing a larger reduction than azimuthal splitting. The reason for this larger reduction of statistical uncertainty associated with rotational splitting is due to the fact that fewer random numbers are sampled for the same MC step, because a zero variance quantity, the constant azimuthal angle, is used. Fig. 3 shows the dependence of the relative efficiency of azimuthal and rotational splitting on d, with respect to the simulation with no splitting ($\varepsilon_{azi}/\varepsilon_{no\text{-}sp}$ and $\varepsilon_{rot}/\varepsilon_{no\text{-}sp}$), using the same simulations described for Fig. 2. The larger efficiency of rotational with respect to azimuthal splitting arises from the

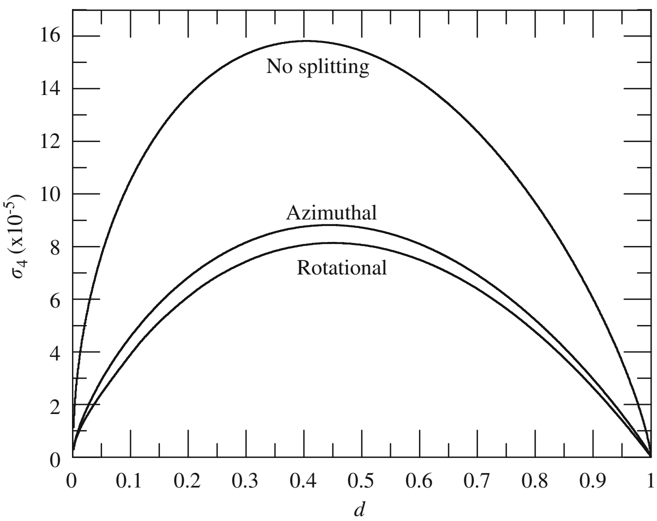

Fig. 2. Statistical uncertainty of the three considered methods as a function of d. Azimuthal and rotational splitting have used a splitting number of 15. Data are plotted using histograms and no smoothing has been applied.

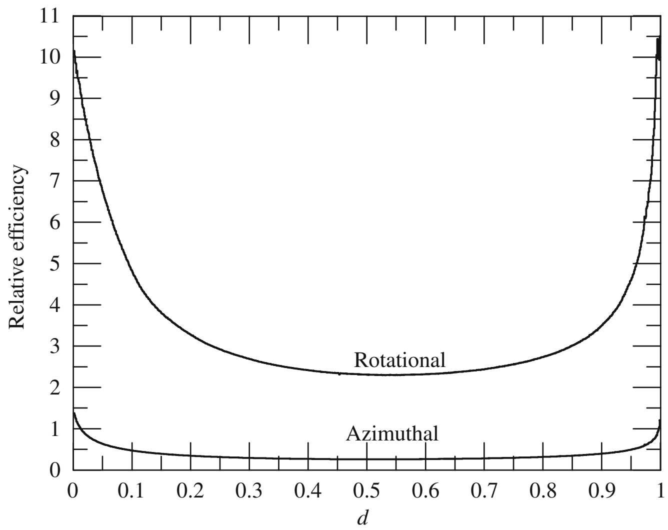

Fig. 3. Relative efficiency of azimuthal and rotational splitting with respect to the simulation with no splitting. The splitting number is 15, while d varies from 0 to 1 in steps of 0.001. Data are plotted using histograms and no smoothing has been applied.

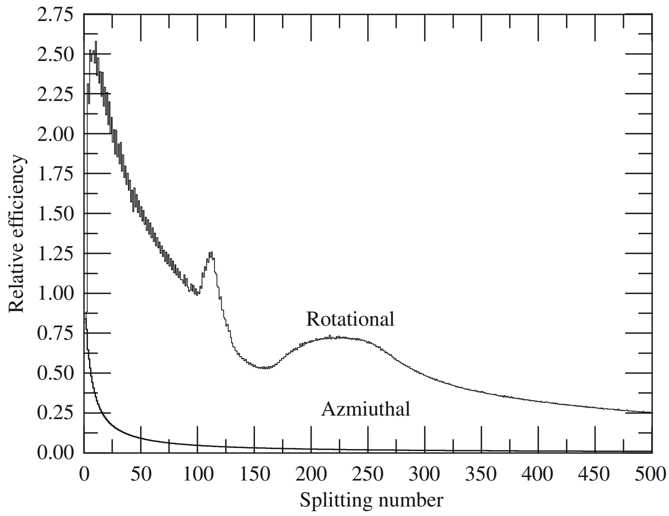

Fig. 4. Relative efficiency of azimuthal and rotational splitting with respect to the simulation with no splitting. The splitting number varies from 1 to 500 in steps of 1, while d equals 0.5. Data are plotted using histograms and no smoothing has been applied.

smaller statistical uncertainty and the smaller number of operations required by the rotational splitting technique.

The efficiency dependence of azimuthal and rotational splitting on the splitting number has been studied by varying the splitting number from 1 (i.e. no splitting) to 500 and setting $d=0.5$. Fig. 4 shows the relative efficiency plots of azimuthal and rotational splitting with varying splitting number. For a splitting number equal to 1 both methods have the same relative efficiency with respect to the no-splitting simulation. With increasing splitting number the relative efficiency of rotational splitting grows rapidly, reaching its maximum for a splitting number equal to 11. For low splitting numbers there are fluctuations in the relative efficiency between two contiguous splitting numbers due to symmetry effects. These effects tend to disappear as the splitting number becomes larger. As to be expected, these effects do not appear in the relative efficiency plot of the azimuthal splitting method, because the random azimuthal angle applied to each split particle eliminates any azimuthal correlation between them. Both methods tend to converge to the same efficiency as the number of split particles increases because the effect of randomly distributing particles becomes less important. The local maxima in the rotational splitting curve (Fig. 4) around splitting numbers 110 and 220 are associated with the particular problem considered.

It can be seen in Figs. 3 and 4 that the efficiency of azimuthal splitting is in most cases smaller than when using no-splitting at all (i.e. relative efficiency is smaller than 1). This is a consequence of the particular problem studied where the number of operations involved for a no-splitting computation is, in many cases, smaller than performing azimuthal splitting. This is not the case in the simulation of a linac, where the transport of particles through the target and beam flattening filter involves a large number of interactions of radiation with matter and therefore less computation time is required for recycling particles than to simulate them through these structural elements of the linac.

4. Simulation efficiency and latent variance in a linac

The efficiency of azimuthal and rotational splitting has been tested by the MC simulation of a Varian Clinac 600C operating in photon mode at 6 MV. The simulations have been performed with the general-purpose radiation transport MC code PENELOPE (Salvat et al., 2008). The geometry file of the linac, as well as the input file, have been generated with the code AutolinaC for automatic generation of PENELOPE simulations of linacs (Brualla et al., 2009b). The validity of the geometry files produced and the simulation parameters employed by AutolinaC have already been tested against experimental data (Brualla et al., 2009b, 2009a; Panettieri et al., 2009). Azimuthal and rotational splitting were applied on a plane located at 24.2 cm from the source, that is, just upstream of the jaws. From the source downstream to this plane the linac exhibits cylindrical symmetry (with the exception of the inclined mirror that due to its very thin profile can be neglected). The splitting number N used with both sampling methods was 15. Apart from the kinds of splitting considered, no other variance-reduction technique was applied. In all linac simulations a PSF was tallied at 65.4 cm from the primary source, that is, at the downstream end of the linac head. The efficiency of each simulation has been evaluated at this plane using the definition of efficiency given in Eq. (5) in terms of the energy of the scored particles.

Two field sizes have been considered, namely $10\times10\,\text{cm}^2$ being a standard field size and $1\times1\,\text{cm}^2$ being hundred times smaller in area. For comparison, simulations in which no splitting has been applied were also run. For the $10\times10\,\text{cm}^2$ field the azimuthal and rotational splitting simulations were more efficient than the simulation without splitting by 7.5% and 9.5%, respectively. In the case of the $1\times1\,\text{cm}^2$ field, the efficiency gain increased to 27% and 36% for the azimuthal and rotational splitting techniques, respectively. In relative terms, rotational splitting is about 30% more efficient than azimuthal splitting for both field sizes.

The latent variance (Sempau et al., 2001) of the PSFs tallied at the downstream end of the linac using a $10\times10\,\text{cm}^2$ field was also studied. It was observed that a simulation run with azimuthal splitting rendered a PSF with a latent variance 14 times smaller than the one obtained using the standard splitting technique with the same splitting number. This reduction in latent variance is similar to the one found by Bush et al. (2007). A PSF tallied with a simulation using rotational splitting yielded a latent variance 40% smaller than the one obtained using azimuthal splitting. The simulations run with standard, azimuthal and rotational splitting used the same splitting number of 15.

5. Conclusion

Using a constant, instead of a random, azimuthal angle for split particles produces a higher efficiency of the azimuthal particle redistribution method, when applying MC simulations of radiation transport in clinical linear accelerators. The method presented here for rotational splitting increases the efficiency of azimuthal splitting by about 30% and reduces the latent variance of phase spaces, tallied using the azimuthal splitting technique, by about 40%. Although the efficiency gains computed for the case of the integration of a segment of a circle are larger than the ones observed for the simulation of an entire linac, there is still a clear improvement of the simulation efficiency of the latter, which helps to perform MC simulations of the whole linac under clinical conditions. It has been shown that the proposed sampling method is particularly useful for simulating small radiation fields, which are of growing interest in radiotherapy.

Acknowledgements

The authors are grateful to F. Salvat (Universitat de Barcelona) and M. Stuschke (Universitätsklinikum Essen) for many stimulating discussions. They are thankful to R. Moss (Joint Research Centre of the European Commission, Petten, NL) for thoroughly revising the manuscript. Financial support is acknowledged from the Sixth Framework Programme of the European Commission through the MAESTRO project (IP CE503564), the Spanish Junta de Comunidades de Castilla-La Mancha (PBC06-0019), Ministerio de Educación y Ciencia (FPA2006-12066) and FEDER.

References

Brualla, L., Palanco-Zamora, R., Wittig, A., Sempau, J., Sauerwein, W., 2009a. Comparison between PENELOPE and electron Monte Carlo simulations of electron fields used in the treatment of the conjunctival lymphoma. Phys. Med. Biol. 54, 5469–5481.

Brualla, L., Salvat, F., Palanco-Zamora, R., 2009b. Efficient Monte Carlo simulation of multileaf collimators using geometry-related variance-reduction techniques. Phys. Med. Biol. 54, 4131–4149.

Bush, K., Zavgorodni, S.F., Beckham, W.A., 2007. Azimuthal particle redistribution for the reduction of latent phase-space variance in Monte Carlo simulations. Phys. Med. Biol. 52, 4345–4360.

Chetty, I.J., Curran, B., Cygler, J.E., et al., 2007. Report of the AAPM task group No. 105: Issues associated with clinical implementation of Monte Carlo-based photon and electron external beam treatment planning. Med. Phys. 34, 4818–4853.

Panettieri, V., Barsoum, P., Wetermark, M., Brualla, L., Lax, I., 2009. AAA and PBC calculation accuracy in the surface build-up region in tangential beam treatments. Phantom and breast case study with the Monte Carlo code PENELOPE. Radiother. Oncol. 93, 94–101.

Reynaert, N., van der Marck, S.C., Schaart, D.R., et al., 2007. Monte Carlo treatment planning for photon and electron beams. Rad. Phys. Chem. 76, 643–686.

Salvat, F., Fernández-Varea, J.M., Sempau, J., 2008. PENELOPE—A code system for Monte Carlo simulation of electron and photon transport. OECD Nuclear Energy Agency, Issy-les-Moulineaux.

Sempau, J., Sánchez-Reyes, A., Salvat, F., ben Tahar, H.O., Jiang, S.B., Fernández-Varea, J.M., 2001. Monte Carlo simulation of electron beams from an accelerator head using PENELOPE. Phys. Med. Biol. 46, 1163–1186.